Math Virtual Learning

Calculus AB

Continuity and One-Sided Limits

April 29, 2020

Calculus AB
Lesson: April 29, 2020

Objective/Learning Target:
Lesson 3 Limits Review
Students will determine continuity and one-sided limits.

Warm-Up:

Note: This is a review of 1st Semester Material. For more examples refer back to your 1st Semester notes.

Click Link then Join and Complete the Slides:

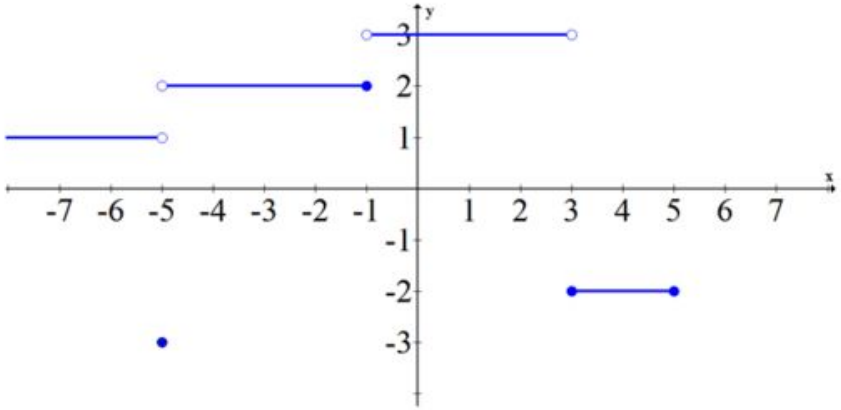
Desmos Limit and Continuity Activity

Watch Videos: Definition of Continuity and One-Sided Limit

One-Sided Limit Example

Examples:

Take the graph below. What are the one sided limits at -5, -1, 3 and 5?



Each point should have two limits, one from the left and one from the right.

$\lim_{x \to -5^-} f(x) = 1$ $\quad$ $\lim_{x \to -5^+} f(x) = 2$

$\lim_{x \to -1^-} f(x) = 2$ $\quad$ $\lim_{x \to -1^+} f(x) = 3$

$\lim_{x \to 3^-} f(x) = 3$ $\quad$ $\lim_{x \to 3^+} f(x) = -2$

$\lim_{x \to 5^-} f(x) = -2$ $\quad$ $\lim_{x \to -5^+} f(x) = DNE$

Examples:

Is the following function continuous?

$$f(x) = \begin{cases} x^2 - 1 & x < -1 \\ 3 & x = -1 \\ -x + 3 & -1 < x \end{cases}$$

Use the definition of continuity.

- $\lim_{x \to 1^-} f(x) = (-1)^2 - 1 = 1 - 1 = 0$
- $f(-1) = 3$
- $\lim_{x \to -1^+} f(x) = -1 + 3 = 2$

$\lim_{x \to a^-} f(x) \neq f(a) \neq \lim_{x \to a^+} f(x)$ so this function is discontinuous at $x = -1$. It is continuous everywhere else.

Practice:

Evaluate the following limits.

1. $\lim_{x \to 3^-} (4x - 3)$
2. $\lim_{x \to 2^+} \left(\frac{1}{x - 2} \right)$
3. $\lim_{x \to 1^+} \left(\frac{x^2 + 2x - 3}{x - 1} \right)$

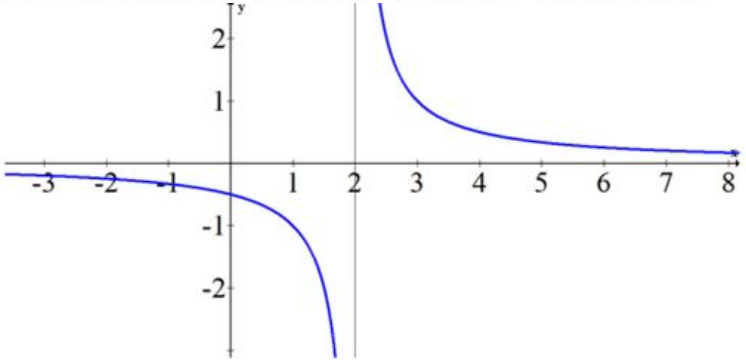
Answer Key:

Once you have completed the problems, check your answers here.

1. $\lim_{x \to 3^-} (4x - 3) = 4 \cdot 3 - 3 = 12 - 3 = 9$

2. $\lim_{x \to 2^+} \left(\frac{1}{x - 2} \right) = DNE$ or ∞

The reason why ∞ is preferable in this case is because the two sides of the limit disagree. One side goes to negative infinity and the other side goes to positive infinity (see the graph below). If you just indicate DNE then you are losing some perfectly good information about the nature of the function.



3. $\lim_{x \to 1^+} \left(\frac{x^2 + 2x - 3}{x - 1} \right) = \lim_{x \to 1^+} \left(\frac{(x - 1)(x + 3)}{(x - 1)} \right) = \lim_{x \to 1^+} (x + 3) = 1 + 3 = 4$

Additional Practice:

Read Section 1.4 in your Calculus book and complete problems 1, 3, 7, 15, 25, 31, 37, and 41

Interactive Practice

More Interactive Practice

Extra Practice with Answers